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# On turbulent drag and heat transfer reduction phenomena and laminar heat transfer enhancement in non-circular duct flow of certain non-Newtonian fluids

M. KOSTIC

Department of Mechanical Engineering, Northern Illinois University, DeKalb, IL 60115, U.S.A.

**Abstract**—The fascinating friction drag and heat transfer reduction phenomena associated with turbulent flows of so-called 'drag-reducing fluids' are not well understood. It is believed that elastic fluid properties are strongly related to these phenomena. However, not all drag-reducing fluids are viscoelastic, nor are all viscoelastic fluids drag-reducing, suggesting that drag reduction and viscoelasticity are probably incidentally accompanying phenomena. Furthermore, the limited research to date has revealed considerable heat transfer enhancement (virtually without friction drag increase) in laminar non-circular duct flows with certain polymer solutions, and has shown that all utilized fluids were indeed viscoelastic! It is argued here that turbulence suppression (i.e. flow laminarization), due to flow-induced anisotropic fluid structure and properties, is a determining factor for the reduction phenomena—not the fluid elasticity—while the latter may be a major cause for the laminar heat transfer augmentation. It is certain that many challenges in this interesting and useful area will keep researchers very busy well into the next century and beyond.

## 1. FASCINATING FLOW AND HEAT TRANSFER BEHAVIOR OF CERTAIN FLUIDS

FEW DISCOVERIES in this century in the area of fluid flow have created such inquisitiveness as the drag-reducing effect of certain additives in common-fluid turbulent flows. Investigators have observed as much as 80% of friction drag reduction in turbulent pipe flow of rather very-dilute solutions (only a fraction of a percentile) of certain additives in water or other common (Newtonian) fluids. When these solutions have been tested in conventional viscometric (laminar) flow, non-Newtonian fluid properties have not been evident within the experimental capability. The density of these dilute solutions was virtually indistinguishable from that of the solvents, to many significant figures. On the basis of such measured phenomenological properties these fluids may have been classified as common, Newtonian fluids. The hydrodynamicists, who have regarded density and viscosity as the only relevent properties of the 'common' fluid flow, have been surprised to find that turbulent flows of such dilute soilutions (with virtually the same phenomenological properties as the solvents) could behave so differently from their Newtonian solvents. Also, no viscoelastic properties, such as the phase shift or the normal stress differences, have been experimentally detected for these dilute solutions. However, further concentration increase of some polymer additives in solvent (e.g. 0.1% and higher) results in pseudoplastic and/or viscoelastic solutions.

Ever since Toms' discovery (1949) [1] that the friction drag of some solutions under *turbulent* flow conditions is *considerably smaller* than the expected values, many researchers have been excited about the peculiar and often unexpected flow and heat transfer behavior of these drag-reducing, so-called viscoelastic fluids. It is now well known that the pressure drop and heat transfer associated with the turbulent duct flow of certain fluids (see Table 1) are considerably lower than the corresponding values for Newtonian fluids. Excellent articles on the subject are presented by Dodge and Metzner [2], Metzner [3], Lumley [4], Virk *et al.* [5], Hoyt [6, 7], Cho and Hartnett [8], and Hartnett [9]. Hence, it is not the intention of this work to review existing literature, but to present the most peculiar behaviors and applications (see Tables 2 and 3), while interested readers are referred to the indicated articles, some of which [3, 6–8] cite extensive references on the subject. Although these 'miraculous' phenomena have been extensively investigated in recent decades, the underlying mechanism producing the drag and heat transfer reduction is not yet fully understood. Not surprisingly, Bird and Curtiss [10] titled their paper *'Fascinating Polymeric Liquids'*, and even the *New York Times* wrote about these unusual and important phenomena in an article "*'Slippery Water' Mystery Seems Finally Solved*" [11]. Actually, the 'mystery' remains in clouds of hypotheses, far from resolution, primarily for two reasons:

(1) the classical isotropic fluid mechanics approach does not work well for the very complex, flow-induced anisotropic fluid structure (even if the corresponding motionless fluid is isotropic); i.e. the constitutive equations are inadequate; and
(2) the turbulence itself is not yet well understood even for 'common' Newtonian fluids.

**NOMENCLATURE**

| | |
|---|---|
| $a, b, c, d$ | constants/coefficients of equation (1) and Table 5 |
| $D_h$ | hydraulic diameter |
| $DI$ | drag increase, equation (3) and Table 6 |
| $DIR$ | drag increase ratio, equation (5) and Table 6 |
| $DR$ | drag reduction, equation (2) and Table 6 |
| $DRR$ | drag reduction ratio, equation (4) and Table 6 |
| $f$ | Fanning friction factor, $\tau_w/\frac{1}{2}\rho U^2$ |
| $Gz$ | Graetz number |
| $H_1$ | uniform wall heat flux axially and constant temperature peripherally, boundary condition |
| $HI$ | heat-transfer increase, equation (3) and Table 6 |
| $HIR$ | heat-transfer increase ratio, equation (5) and Table 6 |
| $HR$ | heat-transfer reduction, equation (2) and Table 6 |
| $HRR$ | heat-transfer reduction ratio, equation (4) and Table 6 |
| $j$ | Colburn heat-transfer factor, $Nu/(Re\, Pr^{1/3})$ |
| $K$ | power-law fluid consistency index, $\tau_w = K\dot{\gamma}^n$ |
| $n$ | power-law fluid power-law index, $\tau_w = K\dot{\gamma}^n$ |
| $Nu$ | Nusselt number |
| $Pr$ | Prandtl number |
| $Ra$ | Rayleigh number |
| $Re$ | Reynolds number |
| $Re^*$ | Kozicki Reynolds number, equation (10) |
| $\bar{t}$ | time constant in Powell–Eyring fluid model, equation (6) |
| $U, \bar{u}$ | time-averaged velocity in main-flow direction |
| $u'$ | fluctuating turbulent velocity component in main-flow direction $x$ |
| $v'$ | fluctuating turbulent velocity component in cross-flow direction $y$ |
| $w'$ | fluctuating turbulent velocity component in cross-flow direction $z$ |
| $x$ | coordinate in main-flow direction |
| $y, z$ | coordinates in cross-flow directions. |

Greek symbols

| | |
|---|---|
| $\dot{\gamma}$ | shear-rate magnitude |
| $\dot{\gamma}_{i,j}$ | shear-rate tensor |
| $\dot{\gamma}^*$ | dimensionless shear rate, equation (6) |
| $\Delta p_p$ | pressure drop for a given pipe length for polymer-solution flow |
| $\Delta p_s$ | pressure drop for a given pipe length for solvent flow |
| $\eta$ | apparent viscosity |
| $\eta_{i,j}$ | anisotropic viscosity, equation (8) |
| $\eta_0$ | zero-shear-rate viscosity (at very small shear rates) |
| $\eta_\infty$ | infinite-shear-rate viscosity (at very large shear rates) |
| $\eta^*$ | dimensionless viscosity, equation (6) |
| $\rho$ | fluid density |
| $\tau_{i,j}$ | shear stress tensor |
| $\tau_w$ | shear stess at wall. |

Subscripts

| | |
|---|---|
| A | asymptotic drag or heat-transfer reduction |
| L | laminar or extended-laminar (ultimate asymptote) |
| T | turbulent Newtonian (without drag or heat-transfer reduction). |

The so-called 'drag-reducing' fluids are simultaneously even stronger 'heat transfer-reducing' fluids in turbulent flows. The most effective drag- and heat transfer-reducing fluids are aqueous solutions of non-linked, high-molecular-weight polymers such as polyethylene oxide and polyacrylamide (see Table 1).

### 1.1. *Turbulent, non-circular duct flow*

Due to the fact that the greatest portion of the momentum and heat transfer resistance occurs in the narrow sublayer region close to the wall, and the velocity and temperature distributions over most of the cross-sectional area are relatively flat, it has been widely believed that the fully developed turbulent friction factor and heat transfer coefficients for a non-circular duct flow could be determined by using the corresponding circular tube correlations, replacing tube diameter with the hydraulic diameter of a non-circular duct. Such an approximation is used due to the lack of appropriate equations for non-circular ducts. As shown by Jones [12], the use of the hydraulic diameter in the well-established circular tube relations has yielded predictions which deviated from measured values up to 20% for large aspect ratio ducts. To improve the predictions, Jones introduced a generalized Reynolds number which is identical to the Kozicki Reynolds number $Re^*$ (see equation (10)) [13], if evaluated for a Newtonian fluid ($n = 1$). Jones demonstrated that the Newtonian circular tube friction factor predictions gave excellent results (within 5% deviations) for rectangular geometry if the circular tube Reynolds number is replaced by $Re^*$. Kostic and Hartnett [14] extended Jones' approach to non-Newtonian fluids, and their findings have been con-

Table 1. Drag-reducing additives and their properties

| Type of additive | Characteristic properties |
|---|---|
| 1. High-polymers<br>—Polyethylene oxide (the best)<br>—Polyisobutylene (oil-soluble)<br>—Polyacrylamide<br>—Carboxymethylcellulose | The most important features are: macromolecules—high-molecular-weight ($10^6$ or higher), linear structure, with maximum extensivity (up to hundreds of thousands of extended-length-to-width ratio of a macromolecule), excellent solubility. Also some other polyelectrolites are known as drag-reducing agents. |
| 2. Soap and surfactant aggregates | Low-molecular-weight alkali-metal and ammonium soap molecules form aggregates or 'micelles' in long-chains. Non-ionic commercial surfactants are very good drag-reducers. |
| 3. Fibers<br>—Asbestos<br>—Nylon<br>—Wood pulp | Asbestos fibers are extremely long (hair-like). Nylon fibers are shorter (length-to-diameter ratio about 50). Wood pulp suspensions in water reduce turbulent friction. Drag reduction is less in fiber–gas suspensions. |
| 4. Solid/liquid particles<br>—Thoria<br>—Sand and dust particles<br>—Droplets in gases | Turbid rivers flow faster than when clear. Pneumatic systems have higher flow rates when dust-laden than with clean air only. Suspension of thoria in water show drag reduction. Even droplets in gases reduce friction. |
| 5. Other natural sources | Natural gums (like guar), algae and bacteria usually produce copious, high-molecular-weight polysaccharide. |

Principal properties of drag-reducing additives

- *Extended length* and/or *sufficient mass* (inertia) to interfere and suppress turbulent fluctuations, particularly transverse ones.
- *Rigidity* and/or *elasticity* to suppress and absorb turbulent fluctuations.

firmed and utilized by Irvine and Karni [15]. The latest recommendations on the use of different friction factor correlations for the power-law fluids in non-circular duct flow are given by Hartnett and Kostic [16].

On the basis of very limited available results, the heat transfer behavior of viscoelastic fluids in non-circular channels seems similar to that in circular pipes. Measured Nusselt numbers of the viscoelastic aqueous polyacrylamide solutions flowing turbulently

Table 2. Known friction and heat-transfer behavior of drag-reducing fluids

| | Characteristic phenomena |
|---|---|
| 1. Friction factor | Considerable friction drag reduction even for minute concentrations (0.5 p.p.m. of polyethylene oxide in water) gives a friction reduction of 40%, which, with increase of polymer concentration, reaches the limiting asymptotic value up to 80%, i.e. the solution friction drag is only 20% of the pure solvent (usually with higher polymer concentrations). |
| 2. Heat transfer | Even stronger heat-transfer reduction than friction drag reduction; over 90% of corresponding Newtonian values for the limiting asymptotic case. Rarely, this phenomenon is useful, as in crude-oil pipelines (smaller losses, i.e. lower viscosity at higher temperature). In contrast, heat transfer is increased in boiling and in laminar flow through non-circular ducts. |
| 3. Entrance lengths | Much longer than the corresponding Newtonian values, on the orders of 100 and 500 hydraulic diameters for hydrodynamic and thermal entrance lengths, respectively. |
| 4. Transition to turbulence | Smoother transition from laminar to turbulent flow, as opposed to abrupt transition of Newtonian fluids. Also higher transitional Reynolds number values (much higher than 2000, often 5000 or higher). In some cases the 'onset' of drag-reduction is encountered. |
| 5. Mean velocity profiles | Flatter velocity profiles (in central region) than the solvent alone. That is quite the opposite from the influence of pipe roughness on the profile. |
| 6. Turbulence structure | Fluctuating $v'$ velocity component is reduced, while axial component $u'$ is less affected; though some results are conflicting. Spacing between large-scale slow-streaks is more than doubled, and time between the 'bursts' (fluid lumps) ejected from the wall region is increased ten-fold. |
| 7. Other | Cavitation is of a different character and is often greatly reduced. Extensional flows through porous media (an application in enhanced-oil-recovery) and jet flows have different characteristics than in pure solvent. Several other behaviors of more-concentrated polymer solutions, such as die-swell, Weissenberg rod-climbing effect, tubeless siphon, inverse secondary flow, etc. are markedly different from Newtonian flows. |

Table 3. Applications with drag-reducing fluids

| | |
|---|---|
| 1. Pipelines | The first application of drag-reducers was use of guar in oil-well 'fracturing', presently a routine practice. Use of oil-soluble polymers in Trans-Alaska Pipeline System is the most impressive success of drag-reducing phenomenon. Typical systems achieve drag reductions of 30–50% at polymer concentrations of 10 p.p.m. (0.001%). Coal and other slurries, and large central-heating hot-water installations are potential candidates, as are many other pipeline systems. |
| 2. Storm-sewer augmentation | By adding polymers the storm-sewer flow capacity may be increased by about 30%. If it is done only during peak demands, such as during storms and floods, it may be very cost-effective, thus eliminating large capital expenditure. |
| 3. Fire-fighting | Water flow rates and jet distances may be increased, or smaller hose-lines may be used. Polymers have been used by fire-fighters in Hamburg (Germany), New York City, and Paris. However, this promising application does not seem to have gained wide acceptance (efficiency is a secondary issue in this case). |
| 4. Ocean/river ship flows | Friction-drag reduction may be obtained by ejecting concentrated polymer solutions near the ship nose, or through a porous wall, or from an ablating coating. Substantial drag-reduction was measured in a water tunnel with ejection of a 500 p.p.m. Polyox solution. Major reduction in polymer cost is necessary for commercial ship application to be feasible. |
| 5. Scientific studies | The friction and heat-transfer reduction phenomena may assist in the studies of molecular properties and, as ironic as it may be (since it changes turbulence structure), in better understanding of turbulence, a very complex flow phenomenon. Several other scientific challenges are emerging. |
| 6. Other | Attractive and possible applications would include: soluble coatings for rapid sinking of oceanographic instruments and other objects; polymer addition to entire city water supply if in high demand due to emergency, such as a major fire; irrigation systems augmentation, etc. |

in a 2:1 rectangular duct show the same general behavior as in the case of turbulent circular pipe flow [17].

1.2. *Laminar, non-circular duct flow*

Contrary to the considerable drag reduction and, unfortunately, even larger heat transfer reduction in turbulent viscoelastic duct flow, the *heat transfer* with more concentrated polymer solutions (around 0.1% and up) in fully developed laminar non-circular duct flow, is surprisingly, *greatly increased*, with virtually *no* friction drag increase. A number of researchers have been thrilled about this heat transfer augmentation, which promises great application potential [18–23]. The observed 200–300% heat transfer augmentation could not be accounted for by the influence of free-convection or temperature-variable fluid properties alone. It is hypothesized that a *non-gravity* secondary flow ought to exist in order to justify the phenomenon.

## 2. EXISTING DRAG REDUCTION THEORIES AND ANOTHER VIEW ON THE PHENOMENA

In light of the large reductions in friction and heat transfer compared to the corresponding turbulent Newtonian flows, it is not surprising that the early reports of these phenomena caused a considerable stir in the scientific community. At first, it appeared that the friction drag reduction phenomenon is miraculous, energy-savior: as if something is obtained from nothing, almost a '*perpetuum mobile*'. However, the friction-drag and heat-transfer reduction phenomena will be enlightened here from a somewhat different point of view, with reference to new, so-called ultimate asymptotes [24]. The existing theories and the present author's hypothesis about possible mechanisms of turbulent drag-reduction are presented in Table 4. The first *Shear Thinning* drag-reduction theory in Table 4 has already been discounted; the second one, based on fluid *Visco-Elasticity and Normal-Stresses*, is the most contradictory and questionable. The remaining theories are inter-related, centering around more or less the same concept: changed (more specifically reduced) turbulence activity/structure of the flow. Therefore, some comments and analysis in that direction are in order.

A logical question arises: 'For a given real/viscous fluid, channel size, and flow-rate, is there the most efficient flow and, if so, what should it be like?'. The answer is rather simple: such flow has to be 'purpose- or goal-oriented'; velocities of all fluid particles have to be exclusively in the main-flow direction, without any components in the other, futile directions (orthogonal to the main-flow or backwards), like turbulent fluctuating components. The former contributes to the over-all flow rate—the goal of the flow—while the latter do not produce any useful flow rate, but rather dissipate energy only, apparently unnecessarily. The most efficient channel flow of any real (viscous) fluid would be the corresponding laminar flow at any Reynolds number value. For such a flow, the friction drag would be the minimum possible, just to overcome the molecular viscous friction. Therefore, the maximum possible turbulent drag reduction would be achieved

Table 4. Theories of reduction phenomena

| Theory | Description |
|---|---|
| 1. Shear Thinning | Originally it was speculated that near-wall-layer, by virtue of shear-thinning may have extremely lower friction coefficient than pure solvent. Later this theory was discounted since it was proved that the shear-thinning friction is somewhat lower, but not nearly that of drag-reduction friction. |
| 2. Visco-Elasticity and Normal-Stresses | This may well be the most unfortunate theory. Drag-reducing polymer solutions are viscoelastic and show the normal-stress differences, but for concentrations extremely high by drag-reduction standards. Very dilute solutions do not exhibit any measurable elasticity, nor change of viscocity from pure solvent, still they are very strong drag reducers. Also, viscoelastic, cross-linked polyacrylic acid (Carbopol) solutions do not show any drag-reduction, except for shear-thinning effect. It may well be that viscoelasticity does not play any major role in drag reduction, but is merely an accompanying property of some drag-reduction fluids. It is known that both viscoelastic and non-elastic fluids may produce drag-reduction. |
| 3. Molecular 'Stretching' | Greatly extended linear macromolecules in shear direction interfere with turbulence, providing a stiffening effect, thus reducing friction drag. Other postulates that molecular entanglements is responsible for interfering with, and enlarging the sublayer eddies. Some have argued that macromolecules' elastic properties and continuous deformation, like a 'yo-yo' effect, are responsible for damping small turbulent eddies, storing and recovering otherwise dissipated turbulent energy. However, for extremely dilute solutions it seems unlikely that such a hypothesis could be valid. |
| 4. Decreased Turbulence Production | Some researchers suggest that polymer additives interfere with the production of turbulence, and that the reduction phenomena are not due to turbulence dissipation but are driven by reduced generation of turbulence. Since the two have to be in balance, their roles may be easily mistaken. |
| 5. Decreased Turbulence Dissipation | It is the belief of the present author that the turbulence energy dissipation via finest eddies are greatly reduced (suppressed) by additives interference, to an extent equal to the drag-reduction, while larger eddies and large-scale flow instability are present (still turbulent flow), but with different and more favorable structure. |
| 6. Vortex Stretching | It is postulated that resistance to vortex stretching reduces the mixing and energy losses. It is further shown that dilute polymer solutions may have thousands of times higher extensional viscosity than the steady-state viscosity, which may have a strong influence on drag-reduction mechanism, believed to play a major role in a region just outside the laminar sublayer ($5 < y^+ < 50$). |
| 7. Non-isotropic Properties and Turbulence | This is another idea of the present author and is elaborated on in the text. Since viscosity is shear-rate dependent and the shear-rate is directional, the solution structure becomes anisotropic; hence viscosity (including dynamic and higher-order stress coefficients) has to be anisotropic: for shear thinning fluids, it is lower in the flow direction and higher in cross-flow directions, thus suppressing considerably the cross-flow fluctuating velocity components (especially small-scale eddy fluctuations). |
| 8. Laminarization of Turbulent Flow | Turbulence is the 'wasteful' dissipation of fluid energy via the finest turbulent eddies, thus it directly increases friction drag. Therefore, drag reduction is a direct measure of partial flow laminarization. By definition, turbulence implies random fluctuations and energy dissipation, otherwise flow instability will have some orderly secondary (and unsteady) flow patterns. |

Unanswered questions:

- Does *viscoelasticity* have any direct relation with turbulent drag-reduction?
- *Influence of wall* may or may not be crucial since polymers may profoundly modify jets and free turbulence?
- *Internal and external boundary layers* may have different influence on drag reduction and an attempt to unify the phenomena may be deceptive?
- Why is *"Onset" of drag-reduction* present with some, but not all drag-reducing fluids?
- Why do additives produce the *maximum friction and heat-transfer reduction asymptotes*, but cannot fully laminarize flow (Ultimate Drag Reduction)?
- Why is the *asymptotic heat-transfer reduction* stronger and occurs for higher polymer concentration than friction drag?... and many other questions!

if all turbulent fluctuating velocity components are suppressed, or never allowed to develop: if the flow is somehow maintained laminar, regardless of the Reynolds number value. This will establish the 'ultimate' friction-drag asymptote. Incidentally, such flow is possible (though difficult) to achieve if the utmost care is taken to avoid any flow disturbances which will otherwise generate flow instability and turbulence for higher Reynolds numbers [25]. Consequently, if drag reduction is observed to be so large as to produce a friction drag smaller than in the corresponding laminar flow, that would warrant some experimental or some other fundamental error(s), including use of inappropriate values of fluid viscosity.

Likewise, the heat transfer under the condition of extended laminar flow (at any Reynolds number) will be laminar, i.e. the absolute physical minimum of heat transfer for a given flow rate. This establishes the 'ultimate' heat transfer asymptote for any turbulent heat-transfer reduction process. The terms 'extended' laminar flow and corresponding 'ultimate' friction-drag and heat-transfer asymptotes refer here to the pertinent laminar friction factor $f_L$ and heat-transfer factor $j_L$, respectively, regardless of the magnitude of the Reynolds number: as if the flow is laminar no matter what! These ultimate asymptotes are presented in Fig. 1, together with other characteristic results for Newtonian (subscript T) and asymptotic drag-reducing turbulent flows (subscript A).

The so-called 'drag-reducing' fluids, like certain polymer solutions, show considerable friction-drag and even stronger heat-transfer reduction as compared to common (Newtonian) fluids. The friction-drag reduction increases with an increase of polymer concentration up to a certain asymptotic limit, first observed by Virk [5]. Similarly, there is an asymptotic limit (higher polymer concentration than for friction-drag asymptote) for heat-transfer reduction [8]. Once these maximum friction-drag or heat-transfer asymptotes have been reached, further increase in polymer concentration does not influence the friction or heat transfer coefficients. It is believed that these maximum asymptotes are functions of the Reynolds number only, and are independent of pipe size and polymer type, which is not the case for the intermediate (non-asymptotic) drag-reducing flows. The maximum asymptotic Fanning friction factor ($f = \tau_w/\frac{1}{2}\rho U^2$) and the Colburn heat-transfer factor ($j = Nu/Re\,Pr^{1/3}$) may be approximately expressed, for the Reynolds number range of interest, by the familiar and simple equations [26]:

$$f = a\,Re^b \quad \text{and} \quad j = c\,Re^d \qquad (1)$$

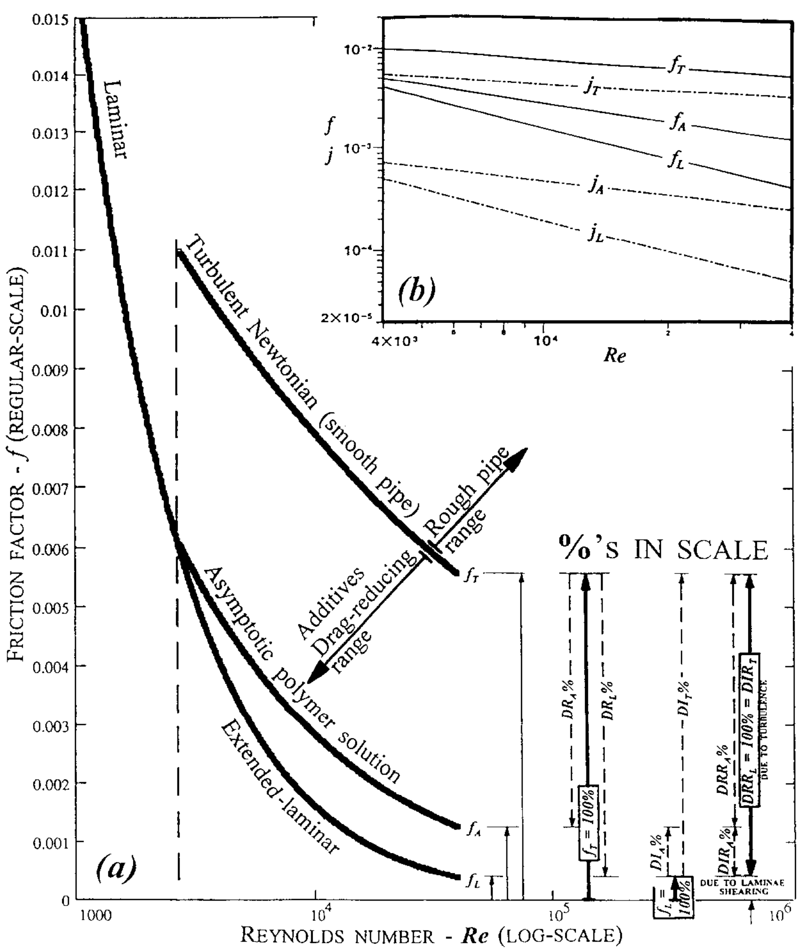


FIG. 1. (a) Friction factors of three characteristic flows (in semi-log scale) and presentation of drag reduction, increase, and ratio terms (see Tables 5 and 6); (b) friction factors—$f$ and heat-transfer $j$-factors for laminar (L), asymptotic (A), and turbulent (T) flows (see Tables 5 and 6).

Table 5. Friction factor and heat transfer $j$-factor equations for different flows

| Equations | | $f = a \cdot Re^b$ | | $j = c(Pr) \cdot Re^d$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | $c$ | | | |
| Flow | Coeff. | $a$ | $b$ | $c(Pr)$ | $Pr = 1$ | 10 | 100 | $d$ |
| Laminar (extended) (L) | | 16.00 | −1.00 | $4.364 \cdot Pr^{1/3}$ | 4.364 | 2.03 | 0.94 | −1.00 |
| Asymptotic-Virk/UIC (A) | | 0.59 | −0.58 | 0.03 | 0.030 | 0.03 | 0.03 | −0.45 |
| Turbulent–Blasius/Newtonian (T) | | 0.079 | −0.25 | $0.023 \cdot Pr^{0.0667}$ | 0.023 | 0.027 | 0.031 | −0.20 |

where the constants $a$, $b$, $c$, $d$ are given in Table 5, together with corresponding constants for ('extended') laminar, and turbulent flow for Newtonian fluids (the Blasius equation). If the conventional Reynolds number is replaced with more generalized, Metzner or Kozicki Reynolds number, equation (1) will be valid for non-Newtonian fluids and/or non-circular ducts, respectively [27].

Originally [7], the drag reduction ($DR$) was defined as pressure drop difference between the solvent (s) and polymer solution (p) with regard to that of the solvent, $DR = 100\%(\Delta p_s - \Delta p_p)/\Delta p_s$, for a given pipe length. However, it is more general to express the drag reduction through the corresponding dimensionless friction factors; and to express similarly the heat-transfer reduction ($HR$) through the dimensionless heat-transfer factors, i.e.

$$DR = \frac{f_T - f}{f_T} \quad \text{and} \quad HR = \frac{j_T - j}{j_T} \tag{2}$$

where non-subscripted factors refer to 'drag-reducing' fluid, while subscript 'T' refers to the corresponding reference, turbulent Newtonian value, without friction-drag or heat-transfer reduction. There is one difference between the original pressure-drop drag reduction and the friction-factor drag reduction. The original drag reduction, defined through pressure drops, is based on a constant flow rate (more practical), while the drag reduction, defined through friction factors, is based on a constant Reynolds number (more fundamental). With the increase of polymer concentration (and solution viscosity) while keeping the flow rate constant, the dimensional pressure drop may start increasing after some polymer concentration level, which is not the case with the dimensionless friction factor at a constant Reynolds number (which requires an increase of flow rate). For very dilute solutions (with a viscosity equal to that of solvent), the drag reductions defined through the pressure drops or friction factors are the same.

As has been pointed out above, it is fundamentally beneficial to analyse friction-drag and heat transfer phenomena with regard to the corresponding ultimate asymptotic values, i.e. the corresponding 'extended' laminar values, as new reference. Since the turbulent friction and heat-transfer factors of drag-reducing fluids are higher than the corresponding extended-laminar values, the new terms, friction-Drag Increase ($DI$) and Heat-transfer Increase ($HI$) are defined, i.e.

$$DI = \frac{f - f_L}{f_L} \quad \text{and} \quad HI = \frac{j - j_L}{j_L} \tag{3}$$

where non-subscripted factors refer to 'drag-reducing' fluid, while subscript 'L' factors refer to the corresponding reference—extended-laminar values—the most efficient flow possible for a real (viscous) fluid, the ultimate asymptotes.

The characteristic asymptotic friction and heat transfer factors for 'drag-reducing' fluids, together with the two reference results (extended-laminar and Newtonian-turbulent flows) are calculated, using equation (1) and Table 5, and presented in Table 6 and Fig. 1. Also, the conventional drag and heat-transfer *reduction* terms of equation (2) and newly defined drag and heat-transfer *increase* terms of equation (3) are calculated for the three characteristic flows in Table 6.

Table 6 reveals that the friction-factor reduction associated with the asymptotic limit of a 'drag-reducing' fluid as compared with the turbulent Newtonian flow, ranges from approximately 52% at $Re = 4000$ to 77% at $Re = 40\,000$, while the corresponding drag-reductions for the hypothetical (but possible) extended-laminar flow (ultimate asymptote) are about 60 and 93%, respectively. It is advantageous to express the asymptotic (or any actual) drag-reduction as the ratio to that of the "ultimate drag reduction" of the extended laminar flow. This ratio, designated here as Drag Reduction Ratio ($DRR$), see Table 6, represents the fundamental drag reduction effectiveness of any drag-reducing additive, i.e. solution. Similarly, the Heat-transfer Reduction Ratio ($HRR$) is defined as the ratio of an actual heat-transfer reduction to that of the extended laminar heat-transfer reduction, the latter being the maximum physical possibility (ultimate asymptote), i.e.

$$DRR = \frac{DR}{DR_L} = \frac{f_T - f}{f_T - f_L} \quad \text{and} \quad HRR = \frac{HR}{HR_L} = \frac{j_T - j}{j_T - j_L} \tag{4}$$

where the nomenclature is explained in equations (2) and (3). It is interesting to note that the heat-transfer reductions of the ultimate asymptotic (extended-laminar) flows, and their ratios, are all larger than their

Table 6. Characteristic values of drag and heat transfer reduction and increase, and their ratios

| | Friction factor | | | Heat transfer $j$-factor ($Pr = 10$) | | |
|---|---|---|---|---|---|---|
| $Re$ | $f_L \times 10^3$ | $f_A \times 10^3$ | $f_T \times 10^3$ | $j_L \times 10^3$ | $j_A \times 10^3$ | $j_T \times 10^3$ |
| 4000 | 4.00 | 4.80 | 9.93 | 0.51 | 0.72 | 5.11 |
| 15000 | 1.07 | 2.23 | 7.14 | 0.14 | 0.40 | 3.92 |
| 40000 | 0.40 | 1.26 | 5.59 | 0.05 | 0.25 | 3.22 |
| | | Drag reduction | | | Heat transfer reduction | |
| $Re$ | $DR_A$ | $DR_L$ | $\mathbf{DRR_A}$ | $HR_A$ | $HR_L$ | $\mathbf{HRR_A}$ |
| 4000 | 0.516 | 0.597 | **0.864** | 0.859 | 0.901 | **0.954** |
| 15000 | 0.687 | 0.851 | **0.808** | 0.899 | 0.966 | **0.931** |
| 40000 | 0.774 | 0.928 | **0.833** | 0.921 | 0.984 | **0.936** |
| | | Drag increase | | | Heat transfer increase | |
| $Re$ | $DI_A$ | $DI_T$ | $\mathbf{DIR_A}$ | $HI_A$ | $HI_T$ | $\mathbf{HIR_A}$ |
| 4000 | 0.201 | 1.483 | **0.136** | 0.418 | 9.08 | **0.046** |
| 15000 | 1.093 | 5.692 | **0.192** | 1.934 | 28.02 | **0.069** |
| 40000 | 2.159 | 12.97 | **0.167** | 4.032 | 62.61 | **0.064** |

drag-reduction counterparts. The reduction ratios, see results in bold in Table 6, are in the 80's and 90's percentiles for the asymptotic friction and heat transfer reduction, respectively.

It is advantageous to analyse the friction-drag and heat-transfer reduction with regard to the most effective flow possible, the extended-laminar flow, for which the reductions are ultimate, i.e. 100%. With regard to this new reference, the friction and heat-transfer factors of drag-reducing fluids are now larger (including the maximum asymptotic values) than the corresponding extended-laminar results. The 'wonder miracle' of drag-reduction is unveiled now in a different, more realistic form: the turbulent flow of these 'miraculous' fluids is actually flow-inefficient ('unnecessarily' energy dissipative), though not nearly as inefficient (dissipative, wasteful) as the common Newtonian fluids. The measure of flow inefficiency or turbulent dissipation of energy is expressed by the newly defined friction Drag Increase ($DI$) and Heat-transfer Increase ($HI$) in equation (3). Their respective Drag Increase Ratio ($DIR$) and Heat-transfer Increase Ratio ($HIR$) are

$$DIR = \frac{DI}{DI_T} = \frac{f-f_L}{f_T-f_L} \quad \text{and} \quad HIR = \frac{HI}{HI_T} = \frac{j-j_L}{j_T-j_L}. \tag{5}$$

In light of the new reference, the extended-laminar flow, the characteristic results are presented in the lower part of Table 6. The physical meaning of the Table 6 last-row numbers is: for an asymptotic drag-reducing flow at 40000 Reynolds number ($Re$), the drag increase is 2.159, or about an additional 216% with reference to the corresponding (same $Re$) laminar flow. Still, this considerable friction-drag increase is much less wasteful than the common Newtonian flow's friction-drag, being an additional 12.97-times larger, or about 1300% more than the reference laminar flow at the same $Re$. These figures yield a drag increase ratio of 0.167, meaning that the inefficiency of the asymptotic drag-reducing flow is only 16.7% of the inefficiency of Newtonian turbulent flow at the same Reynolds number. The heat transfer increases for the asymptotic and turbulent Newtonian flows, in absolute measures of the reference laminar heat transfer, are higher for an additional 4.032- and 62.61-times, respectively. Therefore, the heat-transfer increase ratio is only 0.064, i.e. the asymptotic heat-transfer increase, in addition to the laminar heat-transfer, is only about 6% of the usual heat-transfer increase associated with the common Newtonian turbulent flow, a value considerably smaller than its drag reduction counterpart (16.7%). This 'upside-down' analysis, quite contrary to the conventional drag and heat-transfer reduction analysis, may give an additional insight into the drag and heat-transfer reduction phenomena.

## 3. FLOW-INDUCED NON-ISOTROPIC PROPERTIES: AN ESSENTIAL FLUID MODEL

A high-molecular-weight polymer dissolved in water (or other solvent) builds up a long, macromolecular chain structure, similar to fiber-like composite substance, a sort of flexible (and partially elastic) 'molecular web', which reinforces the original solvent structure [28]. This is reflected in a general increase in the fluid-solution viscosity at all shear rates, particularly at the lower shear rates (see Fig. 2). The important difference between a solid fiber-composite and a polymer-solution is that the fibers are molded (fixed) in the matrix, while the macromolecules may move within the solvent during a flow. The latter will change the original solution structure and make the viscosity (resistance to flow) shear-rate dependent, i.e. the solution becomes the non-Newtonian fluid.

The fluid viscosity is measured in the so-called isometric flow, where only one component of the shear-stress tensor is non-zero, such as in one-dimensional flow through a circular pipe, between cone-and-plate

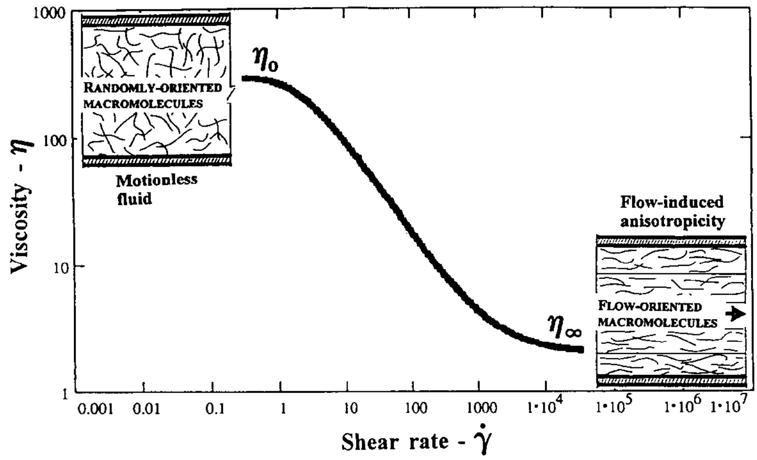


FIG. 2. Shear-rate dependent viscosity and flow-induced anisotropic structure of an aqueous polymer solution.

or concentric cylinders. Under shearing stresses the flexible macromolecular chains, originally randomly shaped and oriented in a motionless fluid, realign along the shearing layers (see Fig. 2). With the shear-rate increase, the macromolecular chains become better aligned and untangled in the flow direction (along iso-strain lines); and the original resistance to flow (zero-shear-rate viscosity, $\eta_0$), due to random shape and orientation of the chains, is considerably reduced, reaching a minimum (infinite-shear-rate viscosity, $\eta_\infty$) at some limiting shear-rate after which further alignment and/or entanglement of the macromolecules is not possible (see Fig. 2). Therefore, $\eta_0 > \eta_\infty > \eta_s$, the latter being the viscosity of the original solvent; these differences are negligible for a very dilute solution ($\eta_0 \approx \eta_\infty \approx \eta_s \approx$ constant). The viscosity–shear-rate function (relationship) is successfully expressed by the Powell–Eyring model

$$\eta^* = \frac{\eta-\eta_\infty}{\eta_0-\eta_\infty} = \frac{\sinh^{-1}(\bar{t}\dot{\gamma})}{(\bar{t}\dot{\gamma})} = \frac{\sinh^{-1}\dot{\gamma}^*}{\dot{\gamma}^*} = \eta^*(\dot{\gamma}^*) \tag{6}$$

where dimensionless viscosity $\eta^*$ and dimensionless shear-rate $\gamma^*$ are defined in equation (6). In dimensionless form $\eta^* = \eta^*(\gamma^*)$, the Powell–Eyring equation is a universal one for any fluid, after the dimensional equation is scaled with the time constant ($\bar{t}$), and the zero- and infinite-shear-rate viscosities ($\eta_0$, $\eta_\infty$), which are characteristic constants of a particular fluid. For example, using equation (6), from the measured steady shear-rate viscosity as function of the shear rate (see Fig. 2), the constants $\bar{t}$, $\eta_0$, $\eta_\infty$ are determined to be 6.1 s, 300 cPs, 3 cPs for a 0.1% polyacrylamide aqueous solutions, respectively [29] (note: 1 centi-Poise [cPs] = 1 mPa·s). For a narrow shear-rate range (often in general for the sake of simplicity), the viscosity may be approximated by the so-called power-law model: $\eta = \eta(\dot{\gamma}) = K\dot{\gamma}^{n-1}$ with two constants, $K$ and $n$ only.

According to this author, the major fallacy is made when the isometric-flow results are generalized (extrapolated) to multi-dimensional flow situations. Namely, the measured directional shear-rate $\dot{\gamma}_{i,j}$ is genralized (substituted) by an invariant shear-rate magnitude $\dot{\gamma}$, of the shear-rate tensor $\{\dot{\gamma}_{i,j} = \partial u_i/\partial x_j + \partial u_j/\partial x_i,\ i \neq j\}$, i.e. through the second invariant of the tensor

$$\dot{\gamma} = \sqrt{(\tfrac{1}{2}(\dot{\gamma}_{i,j}:\dot{\gamma}_{i,j}))}. \tag{7}$$

This implies that the fluid viscosity is directionally independent, i.e. isotropic, and is valid for isotropic fluids only. However, from the observed polymer-solution structure, while under shearing (see Fig. 2), it is obvious that polymer chains will be realigned in a preferable flow direction, making the fluid structure and resistance to flow (i.e. viscosity) directionally dependent, i.e. anisotropic. It should be obvious, at least for such kind of polymer solutions (and probably for many others as well), that resistance to cross-flow (with macromolecules aligned in axial main-flow direction) should be much higher (at least as $\eta_0$) than the resistance to axial flow. The fluid is 'shear-thinned' in one direction, producing flow-induced anisotropic structure and anisotropic viscosity. At least for such fluids, it would be simpler, more realistic, and more accurate (requiring less extrapolation of isometric flow) if the shear rate magnitude $\dot{\gamma}$ of equation (7), used in the viscosity function of equation (6), is expressed as magnitude of the corresponding component of shear-rate tensor: i.e. for the shear-stress $\tau_{i,j}$, the shear-rate magnitude $\dot{\gamma} = |\dot{\gamma}_{i,j}|$. This way, the viscosity will be directionally dependent (anisotropic), i.e. it will be a function of the corresponding shear-rate component

$$\eta_{i,j} = \eta(|\dot{\gamma}_{i,j}|). \tag{8}$$

Indeed, the conventional extrapolation of isometric (one-dimensional) viscosity measurements to multi-dimensional flow situation, using an invariant shear-rate magnitude, made the viscosity shear-rate dependent and non-uniform. However, the obvious flow-induced anisotropicity is overlooked. It is not unfair to state that measured properties in an isometric flow are safely applicable to one-directional flow only, beyond which proper justification is necessary. Furthermore, for a motionless solution (shear-rate is zero for all directions), the so-called zero-shear-rate viscosity will be isotropic, complying with the random orientation of macromolecules. After all, if the fluid structure is isotropic the fluid properties have to be isotropic, and vice versa for an anisotropic fluid structure. The flow-induced anisotropic fluid structure, consequences of which are anisotropic viscosity, dynamic viscosity, and other (higher-order) viscosity and stress coefficients, may play a major role in changing the turbulence structure and other flow patterns, thus influencing the momentum and heat transfer processes.

## 4. ENHANCEMENT OF HEAT TRANSFER IN LAMINAR FLOW OF VISCOELASTIC FLUIDS IN NON-CIRCULAR DUCTS

The fully developed laminar flow of Newtonian fluids in non-circular channels of constant cross-section is a relatively simple flow in the absence of body forces. Furthermore, it is known that the elastic properties of a fluid do not play a significant role with regard to the friction factor and heat transfer in fully developed laminar flow of a viscoelastic fluid through circular ducts. Therefore, the correlations developed for power-law non-Newtonian fluids may be used in that case [3, 8]. In contrast, the fully developed laminar flow of a rheological complex fluid in a non-circular duct turns out to be a complicated flow when accompanied with heat transfer process (non-isothermal flow). The most effective friction factor analysis has been provided by Kozicki *et al.* [13, 30], who generalized the Rabinowitsch–Mooney equation for non-Newtonian fluids, including the special case of the power-law fluid flow in arbitrary but constant cross-section straight duct. They introduced a new Reynolds number by which the friction factors, for the fully developed laminar flow of the power-law fluids and for different ducts, are given by a unique equation:

$$f = \frac{16}{Re^*} \tag{9}$$

$$Re^* = \frac{\rho U^{2-n} D_h^n}{8^{n-1}\left(b^* + \dfrac{a^*}{n}\right)^n K}. \tag{10}$$

The values $a^*$ and $b^*$, dependent on the duct's cross-sectional shape, are given for different aspect ratios of rectangular ducts in ref. [13] and to a fuller extent and more precisely in ref. [27]. For example, the values of $a^*$ and $b^*$ are 0.25 and 0.75 for circular, 0.2121 and 0.6771 for square, 0.2439 and 0.7278 for 5 : 1 aspect ratio rectangular duct, and 0.5 and 1.0 for parallel plates, respectively. The other quantities are defined in the Nomenclature above.

The experimental measurements with non-Newtonian fluids [18–23] reveal that the laminar friction factors are well predicted by Kozicki's correlation for the power-law non-Newtonian fluids (equation (9)), but heat transfer results lie well above the corresponding predictions. Several investigators have suggested that increased heat transfer is probably due to a secondary flow resulting from the viscoelastic behavior of the fluids studied. Such viscoelastic fluids exhibit the so-called normal stress differences that could give rise to increased heat transfer. Such secondary flow has been predicted by Green and Rivlin [31], and there is considerable evidence [32–36] to support its existence. Hartnett and Kostic [18, 22] have reported local heat transfer Nusselt numbers for a rectangular duct of 2 : 1 aspect ratio with the upper and lower walls symmetrically heated. Their results, shown in Fig. 3(a), reveal little difference in heat transfer between the upper and lower walls, and the values are considerably higher than the predictions which allow for the corresponding free convection effects. Furthermore, the influence of free convection and the shear thinning effects are ruled out by previous research, with comparative experiments using Newtonian and non-Newtonian fluids and upper wall heated only to suppress free convection (see Fig. 4) [29]. For example, if upper and bottom walls were heated only, the fully developed Nusselt numbers were 4.8 and 11.4 for the top and bottom wall, respectively, using a Newtonian fluid, while the corresponding values were 14.2 and 17.3 for a polymer solution (see Fig. 4). The difference between the bottom and top wall results is due to free convection effect, but the substantial increase of both values, in particular the top wall value, so much beyond a corresponding forced-convection value of about 5 for the polymer solution, cannot be accounted for by free convection effect alone. There should be a different and more influential cause than the density-driven natural convection, most probably elasticity and/or anisotropicity of the fluid.

It had been reported [33, 35] that non-drag-reducing, polyacrylic acid (Carbopol) aqueous solutions showed even stronger enhancement of heat transfer than the drag-reducing polyacrylamide (Separan) solutions (see Fig. 3(b)). This finding came as a surprise because the Carbopol solutions do not show turbulent drag reduction; hence, they were mistakenly considered as purely viscous non-Newtonian fluids. Consequent measurements of the phase shift using an oscillatory viscometer/rheometer revealed that the Carbopol solutions are indeed viscoelastic. Recently, Gingrich *et al.* [37] have analysed computationally the effect of shear thinning on laminar heat transfer

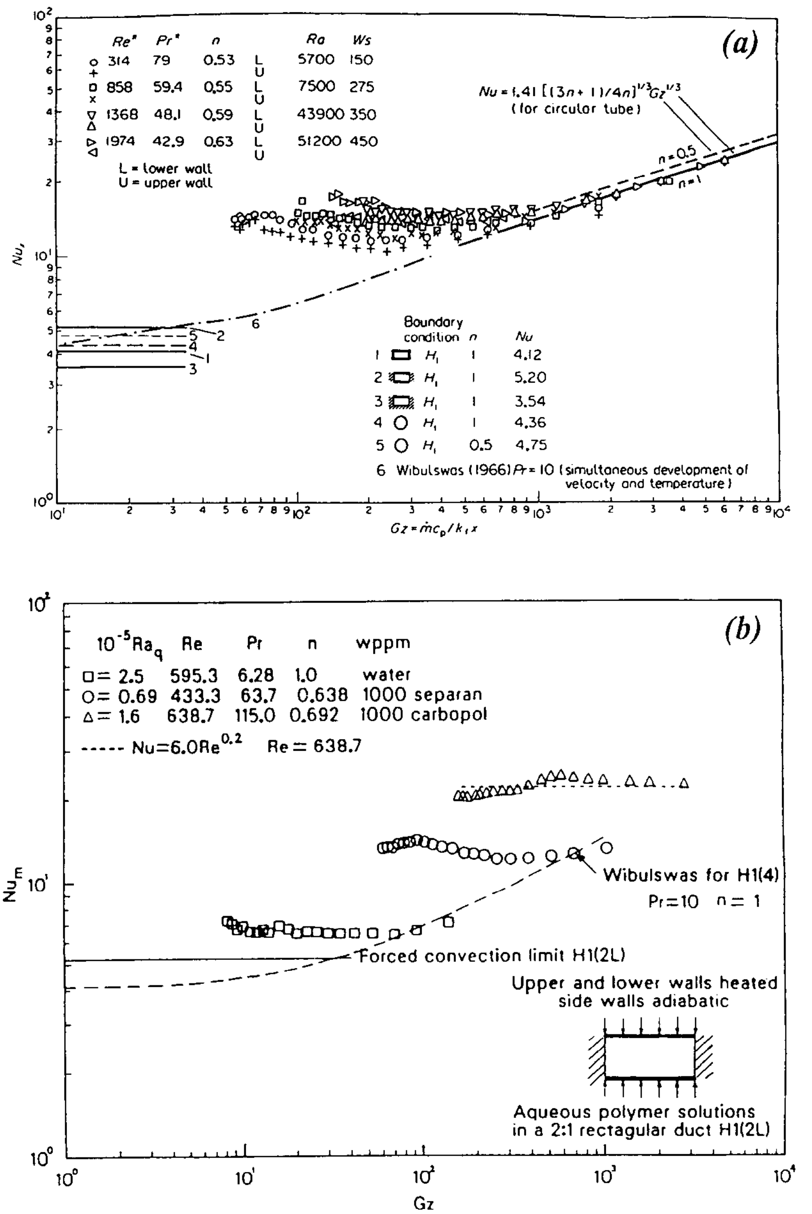


FIG. 3. Local Nusselt numbers vs Graetz numbers for: (a) laminar flow of aqueous polyacrylamide (Separan) solution [18, 29], and (b) aqueous Carbopol solutions [35] in 2 : 1 rectangular duct.

behavior of inelastic non-Newtonian fluid flow in a rectangular duct, and have found substantial heat transfer enhancement (around 100% for non-dissipating flow), though still considerably lower than corresponding viscoelatic fluids' augmentation of 200–300%. Also, Shin *et al.* [38] have found a heat transfer enhancement of 70–80% (over those of constant-property flow) for a high Prandtl number Newtonian fluid flow due to temperature-dependent fluid viscosity only. Nevertheless, the observed 200–300% heat transfer augmentation for viscoelastic non-Newtonian fluid flow in non-circular channels could not be accounted for by influence either of free-convection or of shear-thinning and/or temperature-variable fluid properties alone. Most recently, Payvar [36] has conducted a computational study, taking into account the influence of the normal stress coefficients on heat transfer in laminar flow of viscoelastic fluids in rectangular ducts. His initial findings, quite interestingly, show that a weak secondary flow, driven mainly by the smaller second normal stress coefficient, is responsible for considerable heat transfer enhancement with virtually no increase in friction drag. More and more similar computational studies will be performed in the future, necessitating a need for reliable experimental work.

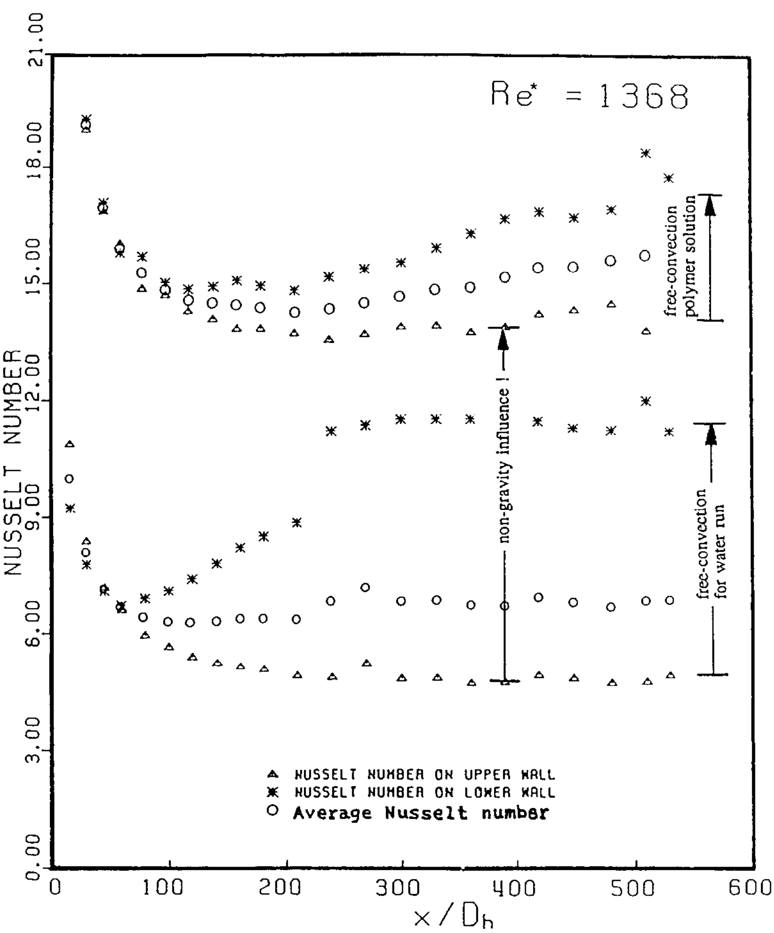


FIG. 4. Measured local Nusselt numbers vs dimensionless axial distance for laminar flow of water and aqueous polyacrylamide (Separan) solution in 2 : 1 rectangular duct [18, 29].

## 5. A CONTRADICTORY ROLE OF FLUID VISCO-ELASTICITY IN DRAG REDUCTION AND SOME (FACTUAL) SPECULATIONS

The fact that the asymptotic friction and heat-transfer factors are much closer to the corresponding extended-laminar results than to the corresponding turbulent Newtonian results, suggests that the addition of polymer (or other additives) decreases the turbulent energy-dissipation, thereby laminarizing the turbulent flow. This is also consistent with the observed high values of the transitional Reynolds number, hydrodynamic- and thermal-entrance lengths.

In addition to the above, this author would like to emphasize the importance of *flow-induced* anisotropic fluid structure and properties. Even though turbulence is a three-dimensional phenomenon, a simplified two-dimensional model is used (see Fig. 5(a)), representing the main-flow fluctuating velocity component, $u'$, and cross-flow component, $v'$, while the third, also cross-flow, component $w'$, having a similar effect as $v'$, is omitted for simplicity. Then, the momentum and heat transfer due to turbulent fluctuating velocity components may be expressed as:

$$\tau_T = -\rho\,\overline{u'v'} \quad q_T = -\rho\, c_p\, \overline{T'v'}. \tag{11}$$

The momentum transfer (i.e. the drag reduction) depends on both fluctuating velocity components, $u'$ and $v'$. The heat transfer (i.e. the heat-transfer reduction) depends on the $v'$ fluctuating velocity component only. Due to anisotropic fluid structure and properties, the resistance to cross-flow ($v'$ direction) is stronger than in the main flow direction, producing non-homogeneous turbulence ($v' \ll u'$). We may pos-

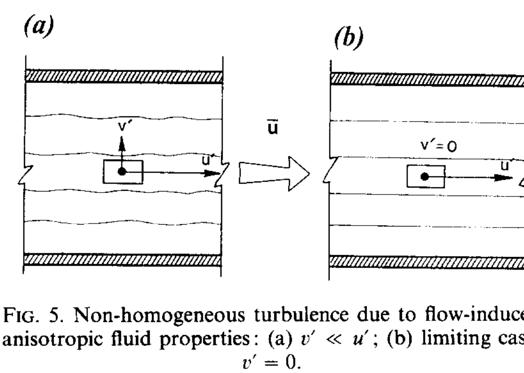


FIG. 5. Non-homogeneous turbulence due to flow-induced anisotropic fluid properties: (a) $v' \ll u'$; (b) limiting case, $v' = 0$.

tulate that the fluctuating turbulent velocities depend on the concentration of macromolecules (or fiber 'threads') in the solvent. With increase of polymer concentration, the mean product of the velocity components $u'v'$ reaches its asymptotic value. This limiting case corresponds to the so-called maximum friction-drag reduction asymptote. By further increase of polymer concentration the mean product of the $T'v'$ will reach its asymptotic value, corresponding to the maximum heat-transfer reduction asymptote. The relative influence of the $u'$ and $v'$ turbulent fluctuations will be clearer if we consider the following limiting case (see Fig. 5(b)). Let us imagine that component $u'$ exists while component $v'$ vanishes completely. In this limiting case there will be no transfer of fluid particles or turbulent heat-transfer in the cross-flow direction. The flow, in the sense of the old Reynolds experiment, will behave as laminar (injected dye will not mix between the layers of main-flow). However, the friction drag (i.e. friction factor) will be somewhat higher than in the laminar flow due to the additional particle shearing (i.e. the back-and-forth fluctuating relative-motion between the layers). Furthermore, the cross-flow turbulent heat transfer depends on the $v'$ fluctuating velocity component only, and for the above limiting turbulence model, the cross-flow heat transfer will be by conduction alone, as in laminar heat transfer. Therefore, it is not surprising that the heat transfer reduction in drag-reducing flows is higher than the reduction in friction drag itself. It has to be noted that the above limiting non-homogeneous turbulence (Fig. 5(b)) cannot occur in reality (the continuity equation is not satisfied), but it does provide a new insight into the physics and trends of actual phenomena when the cross-flow turbulent velocity components $v'$ and $w'$ are much smaller than the main-flow fluctuating component $u'$.

Several up-to-now unanswered questions are raised in Table 4. A number of speculations will be set forth along with some supporting facts.

(1) It is unlikely that fluid *viscoelasticity* plays a major role in turbulent drag reduction phenomena, if at all. Aqueous solutions of polyacrylic acid (Carbopol) are viscoelastic but do not show drag reduction (Carbopol anomaly, but is it?). It is known that non-elastic fluids (including very dilute polymer solutions and some gas suspensions) show considerable drag reduction. It may well be that the two phenomena, drag reduction and viscoelasticity, are independent but accompanying properties of certain fluids.

(2) The *wall influence* is important, but not a determining factor of drag reduction. Drag reduction is associated with certain fluids, not with certain walls or boundaries. Turbulence suppression is present in boundary layer flows as well as in jet and other free turbulence flow situations.

(3) The drag reduction associated with *internal* and *external boundary layers* may be fundamentally different. In external boundary layer flows, the boundary influence and the laminar sublayer are probably predominant. However, in internal boundary layer flows, as in channel flows, the turbulence structure (dissipation) within the enclosed interior determines the overall flow by shaping the sublayer accordingly. The turbulent friction factor is determined by integration of the universal velocity profile over the cross-section, while the contribution of the laminar sublayer may be neglected. In the opinion of this author, the laminar sublayer in internal duct flow is the consequence of internal flow activity, not the other way around. The sublayer simply adjusts to balance the internal stresses, mainly due to overall turbulence in a duct. Attempts to unify the internal and external boundary layer flows may be impossible and therefore deceptive. The similarity between the two universal velocity profiles may be merely due to the coarseness of the log–log scaling.

(4) In some flow situations the so-called *'onset' of drag reduction* is present. In such flows the drag reduction does not start with transition to turbulence, but rather is postponed and occurs at some larger Reynolds number than the transitional. Though, this phenomenon is not fully resolved, the present author agrees with the existing thoughts that some critical shear-stress is needed to realign or untangle the additive 'threads' or macromolecules in the main-flow direction.

(5) Why do the friction and heat-transfer reduction show certain *asymptotic limits*, without achieving the ultimate possible (100%) reduction, i.e. why cannot the additives totally suppress flow instability and turbulence and transfer a high Reynolds number flow in the laminar regime (extended laminar flow)? One possible answer would be: depending on the additive 'thread' properties, the additives interfere only with the smallest turbulent eddies (those most responsible for energy dissipation—friction drag), while larger-scale instabilities and turbulence remain. The former explains the achievement of considerable friction-drag reduction, while the latter justify the turbulence presence and the difficulties in achieving total laminarization. The natural existence of turbulent drag reduction gives us an inspiration to look for additional (possibly artificial) and more efficient additives which, one day, may transfer some existing turbulent flows into ones without turbulence, the extended-laminar flows.

(6) Why is the *asymptotic heat-transfer reduction* stronger than the asymptotic friction-drag reduction? One possible explanation may be a non-homogeneous turbulence, due to the flow-induced anisotropicity of fluid structure and properties. Since the drag reduction depends on the main-flow fluctuating velocity component $u'$, and the cross-flow components $v'$ and $w'$, while the heat-transfer reduction depends on the cross-flow velocity components only. Since the $v'$ and $w'$ components are more suppressed by the flow-induced anisotropic fluid structure than $u'$, that would result in the stronger heat-transfer reduction

than the friction-drag reduction. Another reason may be found by examining the newly defined drag and heat transfer increases for turbulent flow (see $DI_T$ and $HI_T$ values in Table 6). It is interesting that the increase of heat transfer due to turbulence in Newtonian fluids is much higher (five times or more) than corresponding friction drag increase, giving more 'room' for heat-transfer reduction as compared to friction-drag reduction, even for the same level of flow 'laminarization'.

## 6. CONCLUSION

In conclusion, the present author would like to emphasize several points.

(a) If there were not wasteful turbulent energy-dissipation, there could not be any friction-drag reduction. There is not and there could not be room for drag reduction in laminar flow. The new insight is achieved with the above 'upside-down-analysis'. After all, the 'miraculous' drag-reducing fluids/flows are *not* 'energy-savers'—they are just 'not as bad as Newtonian turbulent flows'.

(b) No matter what mechanism is used to describe drag and heat-transfer reduction phenomena, it ultimately results in less turbulent energy dissipation. We need to ask ourselves, 'what is turbulence, after all?'. It is this author's understanding that turbulence is that dissipative flow mechanism which 'breaks and damps' large-scale flow instabilities, considerably increasing the friction drag and heat transfer as compared to laminar flow. Therefore, whether the drag reduction is called molecular or vortex 'stretching', or decreased turbulence production or suppression, in its essence it has to be turbulent flow laminarization. Large-scale instability does exist in the drag-reducing flows, but the most important part of turbulence—the smallest eddies—are considerably eliminated by the additives, and the proof is the substantially reduced friction drag. Therefore, the answer has to be: the reduction phenomena are in essence a turbulent flow laminarization!

(c) This author tends to believe that the determining factor of drag reduction phenomena and turbulent flow laminarization is a 'thread-like' additive structure in solvents, which produce flow-induced anisotropic structure, the consequence of which are anisotropic fluid properties, including but not limited to steady and dynamic viscosities. This is rather obvious for higher concentrations of linear polymer solutions. Since turbulence is the outcome of flow instability, even properties' gradients (and/or the higher order coefficients) may play an important role beyond their absolute values.

(d) The significant (200–300%) heat transfer augmentation in laminar, non-circular duct flows of polymer solutions, without penalty of increased friction drag, is a quite different phenomenon from drag and heat transfer reduction. The laminar heat transfer enhancement is not present with very dilute turbulent drag-reducing solutions, though it is evident in non-drag-reducing, cross-linked polymer solutions, like polyacrylic-acid (Carbopol). On the basis of the well-known analogy of momentum and heat transfer in turbulent flow, one may even suspect the correctness of the observed phenomenon of manifold heat transfer augmentation with virtually no friction drag increase. However, the considerable heat transfer enhancement is observed in laminar non-circular duct flow, where the friction drag is primarily due to the main axial flow. This author hypothesizes that a transverse/circumferential secondary flow ought to exist, though of small magnitude with regard to the main flow, so that friction drag is hardly increased. However, such a weak secondary flow may considerably enhance the heat transfer, which is in transverse direction and is otherwise mainly due to heat conduction through the fluid laminae. The secondary flow's influence on significant heat transfer augmentation is also enhanced by the high Prandtl number value of these fluids. Due to the complex, viscoelastic, and anisotropic properties of these fluids, such a secondary flow hypothesis must be validated by careful experimental measurements of patterns and magnitudes of such (non-isothermal) flows.

In summary, not all drag-reducing fluids are viscoelastic (very dilute solutions, solid–gas suspensions), nor are all viscoelastic fluids drag-reducing (Carbopol solutions). This suggests that the drag reduction and viscoelasticity are probably incidentally accompanying phenomena. However, considerable heat transfer enhancement in laminar non-circular duct flow, discovered in the limited research performed to date, has shown that all utilized fluids were indeed viscoelastic. Further research in this area may help in establishing design methods for a wide range of equipment handling the non-Newtonian and/or viscoelastic fluids, which are very common in the chemical, pharmaceutical, biomedical and food-processing industries. New advancements in electronic equipment cooling, compact heat exchangers, and other areas of rheological and electro-rheological fluids application are also possible.

One more thought on these very complex fluids and even more complex flow phenomena: the phenomenological determination of classical rheological properties (viscosity function and higher stress coefficients) is certainly insufficient to characterize the behavior of very dilute drag-reducing fluids. Flow-induced anisotropicity associated with the small-scale or microscopic fluid structure, is most probably the determining influence of drag and heat transfer reduction, not the fluid visco-elasticity. However, in laminar non-circular duct flow, the rheological (phenomenological) fluid properties, including anisotropic viscosity and elasticity, are responsible for enhanced flow and corresponding heat transfer phenomena. Many more authentic questions may be raised, and only some answers may be ventured on the basis of limited facts. However, this author also believes that nature usually works much simpler than what we sometimes presume. Numerous diverse fluids and flow situations only add to the confusion surrounding these complex phenomena. It is certain that many challenges in this interesting and useful area will keep researchers very busy well into the next century and beyond.

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